Set Theory Basic

1. Define Proper subset . Give an example
2. Define Power set of a set
3. If  the power set of  set A contains 256  elements. Find the number of elements in A?
4. Find the power set of (i) ∅ (ii) {∅}
5. State and prove demorgan's law
6. State and prove Distributive Laws on sets
7. Prove the following formulas  :

 (A-B)-(B-C)= A-C,  AU(BUC)= (AUB)U C, AU(B-A) = AUB

1. State the cardinality principle for two events and three events
2. A group of 60 persons, 45 speak Bengali, 28 speak English and all the persons speak at least one language. Find how many people speak both Bengali and English. Draw a Venn diagram
3. In a group of 150 persons, 75  people can speak English and 55 can speak French. How many can speak English only? How many can speak French only and how many can speak both English and French?

Relation-Types of relation

1. Define the following with examples
2. Reflexive relation
3. Symmetric relation.
4. Irreflexive Relation
5. Antisymmetric Relation
6. Transitive Relation
7. Give an example of a relation which is neither symmetric nor antisymmetric
8. Give an example of a relation which symmetric and antisymmetric
9. Give an example of relation which is neither reflexive nor irreflexive on A= {1,2,3}
10. If R is a relation on A= {1, 2, 3}, such that  (a, b) ∈ R iff  a + b is multiple of 2. Find the relation matrix MR. Also find the graph of the relation

Equivalence Relation

1. Discuss the following with examples
2. Equivalence relation
3. Equivalence class
4. Quotient Set
5. Poset
6. Discuss the properties of equivalence class
7. Find the equivalence classes of the  relation  R={ (a,a),(a,b),(b,a), (b,b),(c,c), (c,d),(d,c),(d,d)} on A= {a,b,c,d}  and draw the relation graph
8. Prove that congruence modulo m is an equivalence relation.
9. For a set A = {a,b,c,d}, find the equivalence relation on S which generates the partition {{a, c},{b},{d} of A. Also write the relation matrix MR and draw the graph of relation.
10. Find the equivalence relation that generates the partition {a,b,d,}, {c} and {e} of the set A={a,b,c,d,e}.
11. Show that the relation R is an equivalence relation in the set A = { 1, 2, 3, 4, 5 } given by the relation R = { (a, b): |a-b| is even }. Also write the relation matrix MR and draw the graph of relation.
12. Let R be the relation “congruence modulo 3 on A= {1,2 ,3,4,5,6,7} . Find the equivalence class generated by the relation .

Function

1. Define the following
2. Function
3. Bijective function
4. One-one function
5. Onto function

1. Explain different types of functions with examples?
2. Let  f, g :  R→R  be defined by f(x)= 2x+1 and g(x) = x2  - 2   ;∀x  respectively. Then find    f -1 ,  f o g ,   g o f, and  f o f.
3. Let f and g are functions from R to R  by   f(x) = 2x2 -1  and   g(x)= 3x  + 4. Determine whether f  is invertible. Find (fog)
4. A = {1,2,3} and f,g,h are functions from A to A given by f = { (1,2), (2,3), (3,1)},  g ={ (1,2), (2,1), (3,3)   and  h= {(1,1), (2,2), (3,1) }  (i)  Find f -1 and  g -1   (ii)  Find (fog)oh  (iii) Show that ( fog) -1  = g -1 of -1
5. Let f and g be two real valued functions, defined by, f(x)= 2x+1 and g(x) = x2  - 2 . Find the value of (f+g)(1)  and  [f- g](2)
6. Check whether the following relations is a function.Justify.
7. f(x) = x2 on R
8. f(x) = √x on R
9. R= {(x,y), x,y Z, x2y =4 }
10. R= {(x,y), x,y R, x<y }

Connectives and truth tables

1. Discuss the connectives with examples and their truth tables.
2. Define tautology and contradiction
3. Find the truth table of following statement formulas. Verify whether its a tautology
4. (p⟶q) ↔(∼q⟶∼p)
5. (PQ)(QR)
6. (p ∨ q) ∧ (¬p) ∧ (¬q)
7. [(p ∧ q) →r] →[p→(q → r)]
8. ∼p ∧ (p⟶q)

Inference Theory

1. Explain the rules of inference
2. Explain Proof by Contradiction
3. Define an argument
4. Check whether true or false
5. ~(p → q) ≡ ~p → ~q
6. p → (q → r), p, ¬r  |−  ¬q
7. p ↔ q ≡ (p ∧ q) ∨ (~ p ∧ ~ q)
8. ⌐ ( p ˄ q)  ≡  (⌐p ˅ ⌐ q)

1. Determine  the Validity of following arguments
2. P ∨ Q, P**→**R, Q**→**S  ⊢ S ∨ R
3. P ∨ Q, Q**→**R, P**→**M, M  ⊢ R ^ (P ∨ Q)
4. P**→**(Q**→**S),R ∨ P , Q  ⊢ R **→**S
5. C ∨D, (C ∨D)**→**~H, ~H**→**(A^~B) and  (A^ ~B)**→**R ∨ S  ⊢R ∨ S
6. H1: If India is country then Kerala is a state , H2 : Kerala is a state ,  C:  India is country
7. Without using truth table show that
8. ∼(p∨q)∨(∼p∧q)≡∼p
9. (⌐P ˄ (⌐Q ˄ R)) ˅ ( Q ˄ R) ˅ (P˄ R) ≡ R.
10. p ↔ q ≡ (p ∧ q) ∨ (~ p ∧ ~ q)
11. Show that (P ^ Q) follow from P ^ Q.
12. Show that the following premises are inconsistent : A**→**B, B**→**C, D**→**C, A^ D

Quantifiers and Truth set

1. Define Universal quantifier
2. Define Existential quantifiers
3. Define truth set of a predicate formula
4. Differentiate between  free and bound variables
5. Express the following sentence in symbolic form using quantifiers –
6. “ Every computer science student must take a discrete mathematics course”.
7. “All men are mortal”
8. “ All  the world loves a lover”.
9. Any integer is either positive or negative
10. Negate the following sentence by writing it in symbolic form using quantifiers –
11. "Every integer is either even or odd".
12. "Every city in India is clean".
13. “All men are mortal”
14. Show that  x [ H(x) M(x) ] H (s)  M (s)
15. Show that  x [ P(x) Q(x) ] , x [ Q(x) R(x) ]     ⊢   x [ P(x) R(x) ]
16. Show that  x [ P(x) Q(x) ] , x [ Q(x) R(x) ]     ⊢   x [ P(x) R(x) ]
17. Show that  x [ H(x) M(x) ]   x,H (x)  x, M(x)
18. Show that  ∀ x (P(x) →Q(x)), ∀x (R(x) → ~Q(x)) ⇒ ∀ x (R(x)→ ~ P(x))
19. Show that Ǝ x, Q(x)  follows logically from   ∀ x (P(x) → Q(x)), and  Ǝ x, P(x)

Correlation & Regression

1. Define the following
2. Correlation
3. Regression
4. Scatter diagram
5. Perfect Correlation
6. Linear and Non Linear Correlation
7. Linear and Non linear regression

1. Discuss about Karl Pearson's Coefficient of Correlation
2. Explain Spearman's Rank Correlation
3. Discuss the properties of a correlation coefficient
4. Discuss the properties of regression coefficients
5. For the given lines of regression x–y=10 and 3x–5y=9, find regression coefficients and correlation coefficient
6. Explain why there are two regression lines for a data.
7. Find KPCC and  rank correlation coefficient for the following data

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| X | 23 | 24 | 23 | 28 | 27 | 28 | 23 |
| Y | 13 | 12 | 14 | 12 | 10 | 18 | 19 |

Also find the regression equations . Calculate the value of y when x=  30

1. Given that the two regression lines are 4x+5y+30=0and 20x+9y-107=0.  Find **i)** the mean values of x and y  **ii)** Correlation coefficient between x and y **iii)** standard deviation of y if standard deviation of  x is 3.

Probability Theory

1. Define
2. Random Experiment
3. Sample space
4. Independent events
5. Equally likely events
6. Mutually exclusive events
7. Exhaustive Events
8. Conditional Probability
9. State and prove addition theorem of probability
10. State and prove multiplication theorem of probability
11. A coin is thrown 3 times. What is the probability that at least one head is obtained?
12. There are 5 green 7 red balls. Two balls are selected one by one without replacement. Find the probability that first is green and second is red.
13. Let  P ( A  ) =0.5,  P ( B )= 0.3
14. Find P( Aor B) if A and B are mutually exclusive
15. Find P(A or B) if A and B are independent

1. The probability that a student will pass Physics is 3/5, and the probability that he'll pass English is 1/3. If the probability that he'll pass both math and English is 1/8, what is the probability that he'll pass at least one subject?
2. What is the probability of getting 53 Mondays in a leap year
3. The probability that a leap year should have exactly 52 Tuesday
4. Find the probability of getting 53 Sundays in a non-leap year
5. A card is drawn from pack of 52 cards find the probability of
6. Getting a King card
7. Getting ace or spade card
8. getting spade  or red card?
9. State and prove Bayes Theorem
10. The probabilities of X , Y and  Z becoming managers  of a company are 4/9, 2/9 and 1/3  respectively. The probability that  Bonus Scheme will be introduced if X,Y and Z become managers are 3/10 , 1/2    and 4/5 respectively.
11. What is the probability that Bonus Scheme will be introduced     and
12. If the Bonus Scheme has been introduced, what is the probability that X was appointed as the manager
13. Three identical boxes contain two balls each .One has both red ,one has one red and one black and the third has two black balls. A person chooses a box at random and takes out a ball. a)  Find the probability that the selected ball is red,  b)  If the ball is red find the probability that the other ball in the box is also red
14. The chance that doctor A will diagnose disease B correctly is 60%.The chance that the patient will die by his treatment after correct diagnosis is 40% and the chance of death by wrong diagnosis is 70% . A patient of Doctor A who had disease B died, What is the probability that his disease was correctly diagnosed
15. There are 2 boxes, one containing 2 white and 3 black balls and the other containing 4 white  and 3 black balls. One ball is transferred from the first box to the second and a ball is drawn from the second. What is the probability that it is white.
16. There are 4 identical bags kept in a dark room containing 6 red and 4 white balls,7 red and 3 white balls ,4 red and 6 white balls and 2 red and 8 white balls. 1 bag is chosen and a ball is taken from the bag. What is the probability that it is a red ball?  Also, find the probability that the ball is selected is from bag 2, given that it is  red.
17. In a factory, Machine A produces 30% of the output, Machine B produces 25% & Machine C 45%.  1% of the output from machine A is defective and it is 1.2% for B and 2% for C.  An item drawn at random from a day’s output is found to be defective. What is the probability that  it is  produced by  A or B.

1. Define
2. Random variable  b)  Probability density function
3. Probability mass function d) Marginal Distribution
4. Explain a) Binomial distribution. b) Poisson Distribution

              c) Normal Distribution   d) Normal Curve

              e) Standardization of Normal distribution

              f) Properties of normal curve

1. Under what conditions a binomial distribution tends to poisson?
2. Check whether the following functions are pdf or not
3. f(x)= 1/4    for  x=-1

              1/3           x=1

               0     elsewhere

1. f(x)= 1/3             x=-1

                          1/3            x=0

                          1/3             x=5

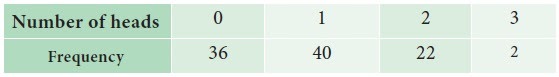
                            0       elsewhere

1. 3.      f(x)= -1/2    for x=1

                            3/2         x=2

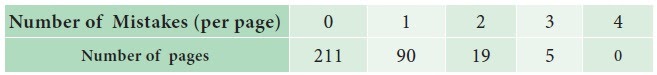
                              0    elsewhere

1. Find the distribution function for the following pdf    : f(x)=x15for x= 1,2,3,4,5 and 0  elsewhere
2. What is the binomial distribution whose mean is 5 and variance 10/3?
3. A set of three similar coins are tossed 100 times with the following results



Fit a binomial distribution and estimate the expected frequencies.

1. The following mistakes per page were observed in a book



Fit a Poisson distribution and estimate the expected frequencies.

1. Given the  following pdf

|  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| f(x) | 0 | c | 2c | 2c | 3c | c2 | 2c2 | 7c2+c |

1. Find the value of c       b)  P(X5) and P(X<3)

1. In a binomial distribution consisting or 5 independent trials, probability of 1 and 2 success are 0.4096 and 0.2048 respectively. The parameter p of the distribution is
2. 4 coins are tossed simultaneously. What is the probability of getting 2 heads
3. The average percentage of failures in an examination is 40. What is the probability that out of a group of 6 candidates  (i) at least 4 passes in the examination (ii) at most 3 failed in the examination
4. If the probability that an item produced by certain machine will be defective is 0.01, find the probability that a random sample of 100 items selected from the total output contains not more than one defective item.
5. In a factory that manufacture razor blades, there is a small chance of  1/500 for each blade to be defective. The blades are in packets of 10.  Use poisson distribution to find out the number of packets containing two defective blades ina consignment of  10,000 packets .
6. In a normal distribution 17% of items are below 30 and 17% of items are above 60. Find the mean and standard deviation  of the normal distribution.
7. In a distribution exactly normal 7% of the items are under 35 and 89% are under 63.What are the mean and standard deviation of the distribution?
8. Find the mean and standard deviation of examination where 44% of the candidates obtained marks below 55 and 6% get above 80 marks.
9. The weekly wages of 1000 workmen are normally distributed around a mean of Rs. 70/- and with a S. D of Rs. 5.  Estimate the lowest wage of 100 highest paid workers.
10. The marks of 500 candidates in an examination with a mean of 45 marks and standard deviation of 20 marks. Given that the pass marks is 41, estimates the number of students who passed the examination?
11. Explain the following
12. Simple random sampling b) Stratified sampling c) Systematic sampling

      d) Cluster Sampling  e) Parameter and Statistic  f)   Sampling Distribution

      g) Standard error    h) Statistical Inference

      i)Point Estimate and Interval estimate j) Central limit theorem

1. Discuss the advantages of sampling over census

1. Explain the following
2. Null and alternate hypothesis
3. Simple and composite hypothesis
4. Parametric and non parametric tests
5. One tail and two tail test
6. Type1 and type 2 errors
7. Significance level
8. Power of test
9. Critical Region
10. Critical value
11. Test statistic
12. Degrees of freedom
13. Explain the test procedure
14. Discuss the test statistic for test of single means
15. Explain the uses of t test  and chi square test
16. Explain the test procedure for
17. Single mean Large sample test
18. Difference of mean- large sample test
19. T test for single mean
20. T test for difference of means
21. Paired t test
22. chi-square test for goodness of fit
23. chi-square test fot testing independence of two attributes
24. A stenographer claims that she can take dictation at the rate of 120 words per minute.Can we reject her claim on the basis of 100 trials in which she demonstrates a mean of 116 words with  standard deviation of 15 words ? Use 5% level of significance
25. A random sample of boots worn by 36 soldiers in a desert region showed an average life of 1.08 years with a standard deviation of 0.6 years.Under the standard conditions, the boots are known to have an average life of 1.28 years.Is there a reason to assert,at 1% level of significance, that use in desert causes the mean life of such boots to decrease ? Assume that the life of  boots is normally distributed.
26. A radio shop sells,on an average,200 radios per day with a standard deviation of 50 radios .After  an extensive advertising campaign, the management will compute the average sales for the next 25 days to see whether an improvement has occurred. Assume that the sales of radios is normally distributed. Test the hypothesis if the average sale for 25 days is found to be 216.
27. An investigation of the relative merits of two kinds of flashlight batteries showed that a random sample of 100 batteries of brand A lasted on average 36.5 hours with a standard deviation of 1.8 hours, while a random sample of 80 batteries of brand B lasted on average of 36.8 hours with a standard deviation of 1.5 hours.Use a level of significance of 0.05 to test whether the observed difference between the average life times is significance.
28. The mean yield of wheat from a district A was 210 lbs with S.D=10 lbs. per acre from a sample of 100 plots. in another district B, the mean yield was 220lbs .with S.D =12 lbs from a sample of 150 plots . Assuming that the standard deviation of the yield in the entire state was 11 lbs , test whether there is any significant difference between the mean yield  of crops in the two districts
29. 12 Rats were given a high protein diet and another set of 7 Rats were given a low protein diet. The gain in weight in grams observed in the 2 sets are given below.

                       High protein diet : 13   14  10  11  12  16  10  8  11  12  9  12

                       Low protein diet :   7    11  10   8  10  13    9

Examine whether the high protein diet is superior to  low protein diet at 5% level of

significance.

1. An IQ test was administered to 5 persons before and after they were trained .The results are given below :

 Candidates https://lh5.googleusercontent.com/OmBB43FBTqOIuHmjA43xJY5_VfiwY9FBR8o4KkygacrJ-kZPribNz933hpu1h4Y0WRyBpZWGAEWzQjrD_Aohj8EjBibxdxcAIR6WYhjkkgP5ZbeLvMz7kE5IoLKV-S2KBCE4UkHe                Ⅰ        II        III        IV        V

 IQ before training             110    120      123      132      125

 IQ after training                120    118      125      136     121

Test whether there is any change in IQ after the training programme.

1. The number of scooter accidents per month in a certain town were as follows: 12, 8, 20, 2, 14, 10, 15, 6, 9, 4. Are these frequencies in agreement with the belief that accident conditions were the same during this 10 month period?
2. In an experiment of breeding pea plants Mendel obtained the following results.   RY =315, WY =  101, RG = 108, WG = 32. Theory predicts that the frequencies should be in the ratio 9: 3: 3: 1. Examine the agreement between theory and observations.
3. The theory predicts the proportion of beans , in the four groups A, B,C, and D should be 9:3:3:1. In an experiment among 1,600 beans the numbers in the four groups were 882, 313, 287 and 118. Does the experimental result support the theory? (The table value of  https://lh4.googleusercontent.com/sDYpZPk6HcpnBk8B1U22JL35aSbiRLKfW_LtiiXl1Mkq4fm0_9DWIbQadTVKskIHwQnhLewF6OivmdsswO48J-N5PE1hbosM297rOHT5YLrLFwLPu52x5kYNZNvdLOf39UePSzEafor 3 d.f. at 5% level of significance is 7.81).
4. From the following data  conclude whether inoculation is effective in preventing tuberculosis.

|  |  |  |
| --- | --- | --- |
|  | Attacked | Not Attacked |
| Inoculated | 31 | 469 |
| Non inoculated | 185 | 1315 |